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Non-singular twisted S-branes from rotating branes

Hong Lü*

George P. and Cynthia W. Mitchell Institute for Fundamental Physics
Texas A & M University, College Station, TX 77843-4242, U.S.A.
E-mail: honglu@umich.edu

Justin F. Vázquez-Poritz†

Department of Physics and Astronomy, University of Kentucky
Lexington, KY 40506, U.S.A., and
Department of Physics, University of Cincinnati
Cincinnati OH 45221-0011, U.S.A.
E-mail: jporitz@pa.uky.edu

Abstract: We show that rotating p-brane solutions admit an analytical continuation to become twisted Sp-branes. Although a rotating p-brane has a naked singularity for large angular momenta, the corresponding S-brane configuration is regular everywhere and exhibits a smooth bounce between two phases of Minkowski spacetime. If the foliating hyperbolic space of the transverse space is of even dimension, such as for the twisted SM5-brane, then for an appropriate choice of parameters the solution smoothly flows from a warped product of two-dimensional de Sitter spacetime, five-dimensional euclidean space and a hyperbolic 4-space in the infinite past to Minkowski spacetime in the infinite future. We also show that non-singular S-Kerr solutions can arise from higher-dimensional Kerr black holes, so long as all (all but one) angular momenta are non-vanishing for even (odd) dimensions.

Keywords: M-Theory, p-branes.

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Contents

1. Introduction

There are two ways of generalizing isotropic extremal p-branes while maintaining the isotropy. One is to introduce a non-extremal factor, which gives rise to non-extremal black p-branes [1, 2]. The other is to relax the condition $dA + \tilde{d}B = 0$ [3]. A nice feature of the second generalization is that the Poincaré symmetry of the p-brane world-volume is maintained. The equations can be reduced to a set of Liouville or Toda equations which can be solved exactly [3]. These solutions can be analytically continued [4] to give rise to cosmological solutions [5, 6, 7, 4], which are now known as S-branes [8].

An interesting feature of the analytical continuation of static black hole or p-brane solutions is that, in many cases, singular solutions become regular. An early example of this is the observation that a Schwarzschild black hole can be analytically continued to a completely regular time-dependent "bubble of nothing" [9, 10]. Another early example is that a four-dimensional de Sitter Kerr black hole can give rise to a smooth four-dimensional Einstein space [11]. This has recently been generalized to five dimensions, which results in an infinite number of five-dimensional inhomogeneous Einstein spaces [12].

It was shown in [13] that singular extremal and non-extremal charged AdS black holes can be analytically continued to regular flux-branes. Also, singular de Sitter charged black holes can be analytically continued to smooth cosmological solutions [13, 14]. The cosmological solution that results in the case of four dimensions can be lifted to a non-singular S-brane configuration in eleven dimensions [15, 16]. Recent examples of non-singular S-branes have also arisen from diholes [17] and Kerr black holes [18, 19]. In this paper, we consider non-singular S-brane configurations which arise from analytically continuing rotating p-branes. For simplicity of terminology, we shall refer to these new solutions as Sp-branes. We also obtain regular S-Kerr solutions in arbitrary dimenensions.

This paper is organized as follows. In sections 2, 3 and 4, we present non-singular SM5, SM2 and SD3-branes, which are related to rotating M5, M2 and D3-branes, respectively. In section 5, we discuss S-Kerr solutions, which arise from higher-dimensional Kerr black holes. In section 6, we present general Sp-branes, which are related to rotating p-branes. Lastly, conclusions are presented in section 7.

2. Non-singular SM5-brane

The rotating M5-brane was constructed in [20]. Adopting the notation of [21], the metric can be written as

$$\begin{aligned} ds_{11}^2 \;=\; & H^{-1/3}\Big(-\Big(1-\frac{2m}{r^3\,\Delta}\Big)dt^2+dx_1^2+\cdots+dx_5^2\Big)+ \\ & +H^{2/3}\Bigg[\frac{\Delta\,dr^2}{H_1\,H_2-\frac{2m}{r^3}}+r^2\Big(d\mu_0^2+\sum_{i=1}^{2}H_i\,(d\mu_i^2+\mu_i^2\,d\phi_i^2)\Big)- \\ & -\frac{4m\,\cosh\alpha}{r^3\,H\,\Delta}\,dt\,\sum_{i=1}^{2}\ell_i\,\mu_i^2\,d\phi_i+\frac{2m}{r^3\,H\,\Delta}\Big(\sum_{i=1}^{2}\ell_i\,\mu_i^2\,d\phi_i\Big)^2\Bigg]\,, \end{aligned} \tag{2.1}$$

where Δ, H and H_i are given by

$$\Delta=H_1\,H_2\Big(\mu_0^2+\frac{\mu_1^2}{H_1}+\frac{\mu_2^2}{H_2}\Big)\,,\qquad H=1+\frac{2m\,\sinh^2\alpha}{r^3\,\Delta}\,,\qquad H_i=1+\frac{\ell_i^2}{r^2}\,. \tag{2.2}$$

The μ_i satisfy $\mu_0^2+\mu_1^2+\mu_2^2=1$. The 4-form field strength is given by $F_{(4)}=*dA_{(6)}$, where

$$A_{(6)}=\frac{1-H^{-1}}{\sinh\alpha}\Big(\cosh\alpha\,dt+\sum_{i=1}^{2}\ell_i\,\mu_i^2\,d\phi_i\Big)\wedge d^5x\,. \tag{2.3}$$

Note that, if one turns off the M5-brane charge by setting $\alpha=0$, the metric becomes the product of the six-dimensional Kerr solution with five-dimensional euclidean space.

We now perform the following analytical continuation:

$$t\to \mathrm{i}\,z\,,\qquad r\to \mathrm{i}\,t\,,\qquad \ell_i\to -\mathrm{i}\,\ell_i\,,\qquad \alpha\to \mathrm{i}\,\alpha\,,\qquad m\to -\mathrm{i}\,m\,, \tag{2.4}$$

and define

$$\mu_0=\tilde\mu_0\,,\qquad \mu_1=\mathrm{i}\,\tilde\mu_1\,,\qquad \mu_2=\mathrm{i}\,\tilde\mu_2\,, \tag{2.5}$$

JHEP07(2004)050

such that $\tilde{\mu}_0^2 - \tilde{\mu}_1^2 - \tilde{\mu}_2^2 = 1$. It is important to note that the $\tilde{\mu}_i$ are unbounded now. The rotating M5-brane becomes a twisted SM5-brane, given by

$$\begin{aligned} ds_{11}^2 &= \tilde{H}^{-1/3}\bigg(\Big(1 - \frac{2m}{t^3\,\tilde{\Delta}}\Big)dz^2 + dx_1^2 + \cdots + dx_5^2\bigg) + \\ &\quad + \tilde{H}^{2/3}\bigg[- \frac{\tilde{\Delta}\,dt^2}{\tilde{H}_1\,\tilde{H}_2 - \frac{2m}{t^3}} + t^2\bigg(- d\tilde{\mu}_0^2 + \sum_{i=1}^2 \tilde{H}_i\,(d\tilde{\mu}_i^2 + \tilde{\mu}_i^2\,d\phi_i^2)\bigg) + \\ &\qquad\qquad + \frac{4m\,\cos\alpha}{t^3\,\tilde{H}\,\tilde{\Delta}}\,dz\,\sum_{i=1}^2 \ell_i\,\tilde{\mu}_i^2\,d\phi_i - \frac{2m}{t^3\,\tilde{H}\,\tilde{\Delta}}\bigg(\sum_{i=1}^2 \ell_i\,\tilde{\mu}_i^2\,d\phi_i\bigg)^2\bigg]\,, \\ A_{(6)} &= \frac{1-\tilde{H}^{-1}}{\sin\alpha}\bigg(\cos\alpha\,dz + \sum_{i=1}^2 \ell_i\,\tilde{\mu}_i^2\,d\phi_i\bigg)\wedge d^5x\,, \end{aligned} \tag{2.6}$$

where

$$\tilde{H} = 1 - \frac{2m\,\sin^2\alpha}{t^3\,\tilde{\Delta}}\,, \qquad \tilde{H}_i = 1 + \frac{\ell_i^2}{t^2}\,, \qquad \tilde{\Delta} = \tilde{H}_1\,\tilde{H}_2\Big(\tilde{\mu}_0^2 - \frac{\tilde{\mu}_1^2}{\tilde{H}_1} - \frac{\tilde{\mu}_2^2}{\tilde{H}_2}\Big)\,. \tag{2.7}$$

Notice that the solution is invariant under $t \to -t$, $m \to -m$.

Unlike the corresponding M5-brane, this S-brane solution does not have an extremal limit. The brane charge vanishes when $m = 0$, in which case the metric is merely that of Minkowski spacetime. Thus, for non-triviality, we consider the case with $m \neq 0$. Another difference between this S-brane and the corresponding M5-brane is that, for the latter solution, one can perform a decoupling limit in which the "1" in the function H can be dropped. The resulting solution can be consistently dimensionally reduced to a seven-dimensional AdS black hole [21]. However, this procedure is not possible for the SM5-brane, for the same reason that an extremal limit is lacking. This feature persists in all of the Sp-branes that we have obtained. This implies that these Sp-branes are intrinsically higher-dimensional and do not arise from the dimensional oxidation of lower-dimensional gauged supergravity solutions.

When all $\ell_i = 0$, the solution is an analytical continuation of the black M5-brane. This is different from the standard S5-brane [4, 8], since in the present case the euclidean symmetry of the worldvolume is broken. Instead, standard S-branes are related by analytical continuation to non-extremal p-branes which have Poincaré symmetry on the worldvolume [4]. Also, note that the presence of the ℓ_i parameters breaks the R-symmetry associated with the isometries of the transverse hyperbolic space. The same story holds for all of the S-brane solutions presented in this paper.

Now let us study the behavior of the metric. In the regions $t \to \pm\infty$, the metric becomes Minkowski spacetime written in the following form:

$$ds_{11}^2 = -dt^2 + t^2 d\Omega_{4,-1}^2 + dz^2 + dx_1^2 + \cdots + dx_5^2\,, \tag{2.8}$$

where $d\Omega_{4,-1}^2 = -d\tilde{\mu}_0^2 + \sum_{i=1}^2 (d\tilde{\mu}_i^2 + \tilde{\mu}_i^2\,d\phi_i^2)$ is the metric of the unit hyperbolic 4-plane. This describes a five-dimensional universe with hyperbolic spatial slices expanding or contracting at a constant rate, together with a stable $\mathbb{E}^5$ or T^5. To analyze the behavior at small t, we first examine the dt^2 term. Since the quantity

$$t^4\tilde{\Delta} = (t^2+\ell_1^2)(t^2+\ell_2^2)\left(1+\frac{\tilde{\mu}_1^2\,\ell_1^2}{t^2+\ell_1^2}+\frac{\tilde{\mu}_2^2\,\ell_2^2}{t^2+\ell_2^2}\right), \tag{2.9}$$

is positive definite for all ℓ_i non-vanishing , we can determine whether t is a time-like or space-like coordinate by examining the function F, given by

$$F = t^4(\tilde{H}_1\,\tilde{H}_2) - 2mt\,. \tag{2.10}$$

This function is positive for $t\to\pm\infty$. If $|m|<m_0$, with m_0 given by

$$m_0 = \frac{\sqrt{6}}{18}\left(\sqrt{\ell_1^4+14\ell_1\ell_2+\ell_2^4}+2\ell_1^2+2\ell_2^2\right)\sqrt{\sqrt{\ell_1^4+14\ell_1\ell_2+\ell_2^4}-\ell_1^2-\ell_2^2}\,, \tag{2.11}$$

then the function F is positive definite and t is always time-like. On the other hand, if $|m|>m_0$, then the function has two positive roots t_+ and t_-. In the regions $t>t_+$ and $t<t_-$, the function F is positive and t is time-like. In the region $t_-<t<t_+$, F is negative and t becomes a space-like coordinate. When $|m|=m_0$, the function F has a second-order zero. In other words, we have $t_+=t_-=t_0$, where t_0 is given by

$$t_0 = \frac{1}{\sqrt{6}}\sqrt{\sqrt{\ell_1^4+14\ell_1\ell_2+\ell_2^4}-\ell_1^2-\ell_2^2}\,. \tag{2.12}$$

Let us first consider the case for which $|m|<m_0$. The twisted SM2-brane is then completely regular, bouncing from one Minkowski spacetime in the infinite past to another Minkowski spacetime in the infinite future. To show this, we note that the function $\tilde{H}$, given by

$$\tilde{H} = \frac{1}{t^4\,\tilde{\Delta}}\left(F + 2m\,t\,\cos^2\alpha + t^4\tilde{H}_1\,\tilde{H}_2\sum_{i=1}^{2}\frac{\tilde{\mu}_1^2\,\ell_i^2}{t^2+\ell_i^2}\right), \tag{2.13}$$

is positive definite if F is non-negative. One might worry that the $2mt\,\cos^2\alpha$ term may cause a problem for negative t. However, $F+2mt\,\cos^2\alpha = t^4\tilde{H}_1\,\tilde{H}_2 - 2mt\,\sin^2\alpha$ is always positive if F is non-negative. Thus, we find that $0<\tilde{H}\le 1$. This implies that the scale factor for the five-dimensional euclidean space $dx^i\,dx^i$ and dz_i^2 remains finite and non-vanishing, while the hyperbolic 4-plane undergoes contraction and expansion.

One important difference between the twisted SM5-brane and the original rotating M5-brane is that $t^4\tilde{\Delta}$ is positive definite for the former case as long as all ℓ_i are non-vanishing. The corresponding term $r^4\Delta$ for the rotating M5-brane always encounters a singularity. This is the reason why the rotating M5-brane with two large angular momentum has a naked singularity while the corresponding SM5-brane is regular.

JHEP07(2004)050

When $|m| > m_0$, the function $\tilde{H}$ may become zero for large $|m|$. This happens when $m \geq m_0/\sin^2\alpha$. The solution then has a naked singularity when $\tilde{H}$ vanishes. For $m_0 < |m| < m_0/\sin^2\alpha$, although F can be negative, $\tilde{H}$ is still positive definite. The solution becomes stationary in the region $t_- < t < t_+$, where $t_\pm$ are the horizons. It is expected that the dz^2 term can change sign in this region, and z is then promoted as a time coordinate. However, the periodic coordinates ϕ_i can also develop a time-like signature. To see this, we note that the determinant of the metric in the z and ϕ_i directions is given by

$$\det(g_{ij}(d\phi_i, dz)) = \tilde{H}\,\tilde{\mu}_1^2\,\tilde{\mu}_2^2\,F\,. \tag{2.14}$$

We see that, when $F < 0$, the signature of either z or one of the ϕ_i is changed. Thus, in the region where $F < 0$ but with positive $(1 - 2m/(t^3\,\tilde{\Delta}))$, one of the compact coordinates ϕ_i will become time-like. To see this more precisely, let us examine, without loss of generality, the $d\phi_1^2$ component of the metric; it is given by

$$\frac{t^2\tilde{H}_1\,(F + 2mt\,\cos^2\alpha) + \tilde{\mu}_1^2\,\ell_1^2\,F + t^4\tilde{H}_1\,\tilde{\mu}_2^2\,\ell_2^2}{t^4\,\tilde{\Delta}\tilde{H}}\,\tilde{\mu}_1^2\,. \tag{2.15}$$

We see that, for $F > 0$, it is positive definite. When F becomes negative then, since $\tilde{\mu}_1^2$ is unbounded, the compact coordinate ϕ_1 can become time-like, giving rise to a closed time-like curve. Since this stationary region is not hidden from external observers by an event horizon, it is not clear if this case has physical relevance.

Now let us consider the case when $|m| = m_0$, for which

$$F(t_0) = 0\,, \qquad F'(t_0) = 0\,. \tag{2.16}$$

In other words, there exists a second-order zero for the function F at t_0. The function F, however, never actually becomes negative. In this case, the Minkowski spacetimes at $t \to \pm\infty$ are disjoint. The point of $t = t_0$ can be viewed as a two-dimensional de Sitter spacetime in the infinite past. To see this, we complete the square for the $d\phi_i$ terms, and the remaining dz^2 metric component is given by

$$\frac{(\tilde{H}_1\,\tilde{H}_2\,t^3 - 2m)\,\tilde{\Delta}\,\tilde{H}^{\frac{2}{3}}}{\tilde{\Delta}(t^3\tilde{H}_1\,\tilde{H}_2 - 2m) + 2\tilde{H}_1\,\tilde{H}_2\,m\,\cos^2\alpha}\,, \tag{2.17}$$

which shrinks to zero at $t = t_0$. In the $t = t_0$ region, the metric for the t and z coordinates becomes

$$ds^2 = t_0^4\,\tilde{\Delta}\,\tilde{H}^{\frac{2}{3}}\left(\frac{(t-t_0)^2\,F''(t_0)}{4t_0^5\,\tilde{H}_1\,\tilde{H}_2\,m\,\cos\alpha}\,dz^2 - \frac{2}{(t-t_0)^2\,F''(t_0)}\,dt^2\right), \tag{2.18}$$

which is two-dimensional de Sitter spacetime with a warp factor $\tilde{\Delta}\,\tilde{H}^{2/3}$ that depends on the coordinates of the four-dimensional hyperbolic space. In the special case $\ell_1 = \ell_2 \equiv \ell$, the conditions (2.11) and (2.12) reduce to

$$m_0 = \frac{8\ell^3}{3\sqrt{3}}\,, \qquad t_0 = \frac{\ell}{\sqrt{3}}\,, \tag{2.19}$$

and the function F simplifies to

$$F = (t - t_0)^2 (t + 2t\,t_0 + 9t_0^2)\,. \tag{2.20}$$

To summarize, for all ℓ_i non-vanishing, m has a critical value m_0. For $|m| < m_0$, the solution is regular everywhere and bounces between two phases of five-dimensional Minkowski spacetime, with a stable $\mathbb{E}^6$ or T^6. If $m = m_0$, then the geometry runs from a warped product of two-dimensional de Sitter spacetime and a hyperbolic 4-plane with a stable $\mathbb{E}^5$ or T^5 to Minkowski spacetime. For $|m| > m_0$, the solution develops closed time-like curves in the stationary region. The solution is further corrupted by a naked singularity when $m \geq m_0/\sin^2\alpha$, due to the contribution of the brane charge.

So far, we have considered the case when both ℓ_i are non-zero. When one or both of them vanish, the solution becomes singular. This is because F will no longer be positive definite and, furthermore, the condition $F(t_0) = 0$ and $F'(t_0) = 0$ can never be satisfied. Thus, there is nothing to prevent the solution passing from the time-like region to the stationary region, where the $t^4\tilde{\Delta}$ term in (2.9) becomes zero at $t = 0$. This yields a naked singularity in the stationary region.

There can be other analytical continuations than what has been presented above. For example, we can take $\mu_0 = \mathrm{i}\,\tilde{\mu}_0$, $\mu_1 = \mathrm{i}\,\tilde{\mu}_1$ and $\mu_2 = \mathrm{i}\,\tilde{\mu}_2$. The reality of the solution now requires ϕ_i, rather than ℓ_i to undergo a Wick rotation. As a result, the functions $\tilde{H}_i$ are given by $\tilde{H}_i = 1 - \ell_i^2/t^2$. It is easy to verify that the solution is not regular.

3. Non-singular SM2-brane

The metric for the rotating M2-brane was obtained in [20]. Adopting the notation of [21], the solution is given by

$$\begin{aligned} ds_{11}^2 = {} & H^{-2/3}\Big(- \Big(1 - \frac{2m}{r^6\,\Delta}\Big) dt^2 + dx_1^2 + dx_2^2 \Big) + \\ & + H^{1/3}\Bigg[\frac{\Delta\, dr^2}{H_1\, H_2\, H_3\, H_4 - \frac{2m}{r^6}} + r^2 \sum_{i=1}^{4} H_i\, (d\mu_i^2 + \mu_i^2\, d\phi_i^2) - \\ & \qquad - \frac{4m\,\cosh\alpha}{r^6\, H\,\Delta}\, dt \Big(\sum_{i=1}^{4} \ell_i\, \mu_i^2\, d\phi_i \Big) + \frac{2m}{r^6\, H\,\Delta} \Big(\sum_{i=1}^{4} \ell_i\, \mu_i^2\, d\phi_i \Big)^2 \Bigg]\,, \end{aligned} \tag{3.1}$$

where Δ, H and H_i are given by

$$\Delta = H_1\, H_2\, H_3\, H_4 \sum_{i=1}^{4} \frac{\mu_i^2}{H_i}\,, \qquad H = 1 + \frac{2m\,\sinh^2\alpha}{r^6\,\Delta}\,, \qquad H_i = 1 + \frac{\ell_i^2}{r^2}\,, \tag{3.2}$$

and the μ_i satisfy $\sum_{i=1}^{4} \mu_i^2 = 1$. The 3-form gauge potential is given by

$$A_{(3)} = \frac{1 - H^{-1}}{\sinh\alpha} \Big(- \cosh\alpha\, dt + \sum_{i=1}^{4} \ell_i\, \mu_i^2\, d\phi_i \Big) \wedge d^2x\,. \tag{3.3}$$

We perform the following analytical continuation:

$$
\begin{array}{llll}
t \to \mathrm{i}\, z\,, & r \to \mathrm{i}\, t\,, & \phi_4 \to \mathrm{i}\, \phi_4\,, & \\
\alpha \to \mathrm{i}\, \alpha\,, & m \to -\mathrm{i}\, m\,, & \ell_j \to -\mathrm{i}\, \ell_j & j = 1, 2, 3\,,
\end{array} \tag{3.4}
$$

and ℓ_4 is unchanged. We define

$$
\mu_4 = \tilde{\mu}_4\,, \qquad \mu_j = \mathrm{i}\, \tilde{\mu}_j\,, \tag{3.5}
$$

such that $\tilde{\mu}_4^2 - \sum_{j=1}^3 \tilde{\mu}_j^2 = 1$. Thus the $\tilde{\mu}_i$ are unbounded.

The rotating M2-brane becomes a twisted SM2-brane given by

$$
\begin{aligned}
ds_{11}^2 &= \tilde{H}^{-2/3} \left(\left(1 - \frac{2m}{t^6\, \tilde{\Delta}}\right) dz^2 + dx_1^2 + dx_2^2 \right) + \\
&\quad + \tilde{H}^{1/3} \Bigg[- \frac{\tilde{\Delta}\, dt^2}{\tilde{H}_1\, \tilde{H}_2\, \tilde{H}_3\, \tilde{H}_4 - \frac{2m}{t^6}} + t^2 \sum_{j=1}^3 \tilde{H}_j\, (d\tilde{\mu}_j^2 + \tilde{\mu}_j^2\, d\phi_j^2) + \\
&\qquad\quad + t^2\, \tilde{H}_4\, (-d\tilde{\mu}_4^2 + \tilde{\mu}_4^2\, d\phi_4^2) + \frac{4m\, \cos\alpha}{t^6\, \tilde{H}\, \tilde{\Delta}}\, dz \left(\sum_{i=1}^4 \ell_i\, \tilde{\mu}_i^2\, d\phi_i \right) - \\
&\qquad\quad - \frac{2m}{t^6\, \tilde{H}\, \tilde{\Delta}} \left(\sum_{i=1}^4 \ell_i\, \tilde{\mu}_i^2\, d\phi_i \right)^2 \Bigg]\,, \\
A_{(3)} &= \frac{1 - \tilde{H}^{-1}}{\sin\alpha} \left(- \cos\alpha\, dz + \sum_{i=1}^4 \ell_i\, \tilde{\mu}_i^2\, d\phi_i \right) \wedge d^2x\,,
\end{aligned} \tag{3.6}
$$

where

$$
\begin{aligned}
\tilde{H} &= 1 - \frac{2m\, \sin^2\alpha}{t^6\, \tilde{\Delta}}\,, \qquad \tilde{\Delta} = \tilde{H}_1\, \tilde{H}_2\, \tilde{H}_3\, \tilde{H}_4 \left(\frac{\tilde{\mu}_4^2}{\tilde{H}_4} - \sum_{j=1}^3 \frac{\tilde{\mu}_j^2}{\tilde{H}_j} \right)\,, \\
H_4 &= 1 - \frac{\ell_4^2}{t^2}\,, \qquad \tilde{H}_j = 1 + \frac{\ell_j^2}{t^2} \qquad j = 1, 2, 3\,.
\end{aligned} \tag{3.7}
$$

This solution is invariant under $t \to -t$.

In the asymptotic $t \to \pm\infty$ regions, the geometry becomes Minkowski spacetime, describing an eight-dimensional universe expanding or contracting at a constant rate, together with a stable $\mathbb{E}^3$ or T^3. At small t, the analysis is similar to the previous section. In order to avoid a singularity when $H_4 = 1 - \ell_i/t^2$ vanishes or becomes negative, it is necessary to set $\ell_4 = 0$. Having done that, the quantity

$$
t^6\, \tilde{\Delta} = (t^2 + \ell_1)^2 (t^2 + \ell_2^2)(t^2 + \ell_3^2) \left(1 + \sum_{i=1}^3 \frac{\tilde{\mu}_i^2\, \ell_i^2}{t^2 + \ell_i^2} \right) \tag{3.8}
$$

is positive definite as long as ℓ_1, ℓ_2 and ℓ_3 are all non-vanishing. The global structure of the metric is now largely determined by the function

$$
F = t^6\, \tilde{H}_1\, \tilde{H}_2\, \tilde{H}_3 - 2m\,. \tag{3.9}
$$

When we have

$$m < m_0 \equiv \frac{1}{2}(\ell_1\,\ell_2\,\ell_3)^2\,, \qquad \ell_4 = 0\,, \tag{3.10}$$

the function F is positive definite. The function $\tilde{H}$, given by

$$\tilde{H} = t^{-6}\tilde{\Delta}^{-1}\left(F + 2m\cos^2\alpha + t^6\tilde{H}_1\,\tilde{H}_2\,\tilde{H}_3\sum_{i=1}^{3}\frac{\tilde{\mu}_i^2\,\ell_i^2}{t^2+\ell_i^2}\right), \tag{3.11}$$

can also be seen to be positive definite. Thus, the solution is regular everywhere, bouncing between two Minkowski spacetimes in the $t \to \pm\infty$ regions. It is worth pointing out that the condition $m < m_0$ includes the possibility of negative m.

When $m \geq m_0$, the function F has two real roots $\pm t_0$, between which the solution becomes stationary. For $m \geq m_0/\sin^2\alpha$, the function $\tilde{H}$ can vanish and, hence, the solution is singular. For $m_0 \leq m < m_0/\sin^2\alpha$, we expect that the solution has no curvature singularity. However, metric develops closed time-like curves. This can be seen from the determinant of the metric for the z and ϕ_i directions, which is given by

$$\det(g_{ij}) = \tilde{H}^{2/3}\,\tilde{\mu}_1^2\,\tilde{\mu}_2^2\,\tilde{\mu}_3^2\,\tilde{\mu}_4^2\,F\,. \tag{3.12}$$

Comparing this with the dz^2 term, it is easy to see that the compact coordinates ϕ_i can develop time-like signatures. Without loss of generality, we can examine the $d\phi_1^2$ term, given by

$$\frac{\tilde{\mu}_1^2\,d\phi_1^2}{t^6\,\tilde{H}\,\tilde{\Delta}}\left[t^2\tilde{H}_1\,(F + 2m\cos^2\alpha) + F\,\tilde{\mu}_1^2\,\ell_1^2 + t^6\tilde{H}_1\,\tilde{H}_2\,\tilde{H}_3\sum_{i=2}^{3}\frac{\tilde{\mu}_i^2\,\ell_i^2}{t^2\tilde{H}_i}\right]. \tag{3.13}$$

The positivity of this term is ensured only for positive F, since $\tilde{\mu}_i$ is unbounded.

To summarize, when $\ell_4 = 0$ and $\ell_j \neq 0$ with $j = 1, 2, 3$, there exists a critical value $m_0 = \frac{1}{2}(\ell_1\,\ell_2\,\ell_3)^2$. For $-\infty < m < m_0$, the solution is regular everywhere, bouncing between two Minkowski spacetimes at $t \to \pm\infty$. The solution is totally symmetric under $t \to -t$. The function $\tilde{H}$ is finite but non-vanishing, which implies the stability of a $\mathbb{E}^3$ (or T^3) portion of the spacetime. For $m > m_0$, the solution develops closed time-like curves. Furthermore, for $m > m_0/\sin^2\alpha$ the solution becomes singular, due to the brane charge contribution. It is worth pointing out that unlike the SM5-brane, here we are forced to set one of the angular momentum , which implies that F never develops a second-order zero. Thus there can be no dS_2 factor arising in this case. This turns out to be a generic feature of twisted S-branes with an odd-dimensional hyperbolic transverse space.

If there are additional ℓ_i vanishing, the solution becomes singular at $t = 0$, which is in a stationary region. There are alternative analytical continuations which lead to singular solutions.

4. Non-singular SD3-brane

The metric for the rotating D3-brane was found in [22, 23]. Adopting the notation of [21], the solution is given by

$$ds_{10}^2 = H^{-1/2}\left(-\left(1 - \frac{2m}{r^4\,\Delta}\right)dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) +$$

$$+H^{1/2}\left[\frac{\Delta\,dr^2}{H_1\,H_2\,H_3-\frac{2m}{r^4}}+r^2\sum_{i=1}^{3}H_i\,(d\mu_i^2+\mu_i^2\,d\phi_i^2)-\right.$$
$$\left.-\frac{4m\,\cosh\alpha}{r^4\,H\,\Delta}\,dt\,\sum_{i=1}^{3}\ell_i\,\mu_i^2\,d\phi_i+\frac{2m}{r^4\,H\,\Delta}\left(\sum_{i=1}^{3}\ell_i\,\mu_i^2\,d\phi_i\right)^2\right], \qquad (4.1)$$

where Δ, H and H_i are given by

$$\Delta=H_1\,H_2\,H_3\sum_{i=1}^{3}\frac{\mu_i^2}{H_i}\,,\qquad H=1+\frac{2m\,\sinh^2\alpha}{r^4\,\Delta}\,,\qquad H_i=1+\frac{\ell_i^2}{r^2}\,, \qquad (4.2)$$

and the μ_i satisfy $\mu_1^2+\mu_2^2+\mu_3^2=1$. The self-dual 5-form field strength is given by $F_{(5)}=dA_{(4)}+{*dA_{(4)}}$, where

$$A_{(4)}=\frac{1-H^{-1}}{\sinh\alpha}\left(-\cosh\alpha\,dt+\sum_{i=1}^{3}\ell_i\,\mu_i^2\,d\phi_i\right)\wedge d^3x\,. \qquad (4.3)$$

We perform the following analytical continuation:

$$t\to\mathrm{i}\,z\,,\qquad r\to\mathrm{i}\,t\,,\qquad \phi_3\to\mathrm{i}\,\phi_3\,,$$
$$\ell_1\to-\mathrm{i}\,\ell_1\,,\qquad \ell_2\to-\mathrm{i}\,\ell_2\,,\qquad \alpha\to\mathrm{i}\,\alpha\,,$$

and ℓ_3 and m are unchanged. We define

$$\mu_1=\mathrm{i}\,\tilde\mu_1\,,\qquad \mu_2=\mathrm{i}\,\tilde\mu_2\,,\qquad \mu_3=\tilde\mu_3\,, \qquad (4.5)$$

such that $\tilde\mu_3^2-\tilde\mu_1^2-\tilde\mu_2^2=1$.

The rotating D3-brane becomes a twisted SD3-brane, given by

$$ds_{10}^2=\tilde H^{-\frac12}\left(\left(1-\frac{2m}{t^4\,\tilde\Delta}\right)dz^2+dx_1^2+dx_2^2+dx_3^2\right)+$$
$$+\tilde H^{\frac12}\left[-\frac{\tilde\Delta\,dt^2}{\tilde H_1\,\tilde H_2\,\tilde H_3-\frac{2m}{t^4}}+t^2\left(\sum_{j=1}^{2}\tilde H_j\,(d\tilde\mu_j^2+\tilde\mu_j^2\,d\phi_j^2)-\tilde H_3(d\tilde\mu_3^2-\tilde\mu_3^2\,d\phi_3^2)\right)+\right.$$
$$\left.+\frac{4m\,\cos\alpha}{t^4\,\tilde H\,\tilde\Delta}\,dz\sum_{i=1}^{3}\ell_i\,\tilde\mu_i^2\,d\phi_i-\frac{2m}{t^4\,\tilde H\,\tilde\Delta}\left(\sum_{i=1}^{3}\ell_i\,\tilde\mu_i^2\,d\phi_i\right)^2\right],$$
$$A_{(4)}=\frac{1-\tilde H^{-1}}{\sin\alpha}\left(-\cos\alpha\,dz+\sum_{i=1}^{3}\ell_i\,\tilde\mu_i^2\,d\phi_i\right)\wedge d^3x\,, \qquad (4.6)$$

where

$$\tilde H=1-\frac{2m\,\sin^2\alpha}{t^4\,\tilde\Delta}\,,\qquad \tilde\Delta=\tilde H_1\,\tilde H_2\,\tilde H_3\left(\frac{\tilde\mu_3^2}{\tilde H_3}-\frac{\tilde\mu_1^2}{\tilde H_1}-\frac{\tilde\mu_2^2}{\tilde H_2}\right),$$
$$\tilde H_3=1-\frac{\ell_3^2}{t^2}\,,\qquad \tilde H_j=1+\frac{\ell_j^2}{t^2}\qquad j=1,2\,. \qquad (4.7)$$

This solution is invariant under $t\to-t$.

JHEP07(2004)050

Since the foliating sphere for D3-brane is odd-dimensional, the properties of the SD3-brane solution are rather analogous to those of the SM2-brane. In the asymptotic $t \to \pm\infty$ regions, the geometry becomes Minkowski spacetime, describing a six-dimensional expanding or contracting universe, together with a stable $\mathbb{E}^4$ or T^4.

For regularity, it is necessary to set ℓ_3 to zero. When the remaining ℓ_1 and ℓ_2 are both non-zero. there exists a critical value $m_0 = frac12(\ell_1\,\ell_2\,\ell_3)^2$. For $-\infty < m < m_0$, the solution is regular everywhere, bouncing between two six-dimensional Minkowski spacetimes at $t \to \pm\infty$ with a stable $\mathbb{E}^4$ (or T^4). The solution is totally symmetric under $t \to -t$. The function $\tilde{H}$ is finite but non-vanishing, implying the stability of the $\mathbb{E}^4$ (or T^4). For $m > m_0$, the solution develops closed time-like curves. Furthermore, for $m > m_0/\sin^2\alpha$, the solution becomes singular due to the brane charge contribution.

If there are additional ℓ_i vanishing, the solution becomes singular at $t = 0$, which is in a stationary region. There exist more ways of taking analytical continuations, all of which lead to singular solutions.

5. S-Kerr solutions

So far, we have considered three examples of rotating p-branes analytically continuing to S-brane configurations. The S-brane solutions can become regular for large angular momenta. One feature is that the introduction of the brane charge does not affect the regularity of the solutions in any significant way. Thus, the discussion of the regularity is more or less the same for S-Kerr solutions, which can be analytically continued from Kerr black holes. Regular S-Kerr solutions for four and five dimensions were obtained in [18, 19]. Higher-dimensional S-Kerr solutions with one angular momentum parameter were obtained in [19] but these solutions are singular. In this section, we obtain regular S-Kerr solutions in arbitrary dimension. The situation differs depending on whether the number of dimensions is even or odd.

5.1 Even dimensions: $D = 2n+2$

The Kerr-Schild solution in $D = 2n+2$ dimensions, with a sphere of $d = 2n$ dimensions, has the metric [24]

$$\begin{aligned} ds_D^2 \;=\; & -dt^2 + \frac{\Delta\, dr^2}{\prod_{i=1}^n H_i - \frac{2m}{r^{2n-1}}} + r^2 d\mu_0^2 + r^2 \sum_{i=1}^n H_i\,(d\mu_i^2 + \mu_i^2\, d\phi_i^2) + \\ & + \frac{2m}{r^{2n-1}\,\Delta}\left(dt - \sum_i^n \ell_i\,\mu_i^2\, d\phi_i\right)^2 , \end{aligned} \tag{5.1}$$

where

$$\mu_0^2 + \sum_{i=1}^n \mu_i^2 = 1\,, \qquad \Delta = \left(\mu_0^2 + \sum_{i=1}^n \frac{\mu_i^2}{H_i}\right)\prod_{i=1}^n H_i\,, \tag{5.2}$$

and $H_i = 1 + \ell_i^2/r^2$. We can now perform the analytical continuation

$$t \to \mathrm{i}\, z\,, \qquad r \to \mathrm{i}\, t\,, \qquad \ell_i \to -\mathrm{i}\,\ell_i\,, \qquad \alpha \to \mathrm{i}\,\alpha\,, \qquad m \to -\mathrm{i}\, m\,, \tag{5.3}$$

together with taking $\mu_0 = \tilde\mu_0$ and $\mu_i = \mathrm{i}\,\tilde\mu_i$, such that $\tilde\mu_0^2 - \sum_{i=1}^n \tilde\mu_i^2 = 1$. The Kerr-Schild solution becomes an S-Kerr solution, given by

$$ds_D^2 = dz^2 - \frac{\tilde\Delta\, dt^2}{\prod_{i=1}^n \tilde H_i - \frac{2m}{t^{2n-1}}} - t^2 d\tilde\mu_0^2 + t^2 \sum_{i=1}^n \tilde H_i\,(d\tilde\mu_i^2 + \tilde\mu_i^2\, d\phi_i^2) - \\ - \frac{2m}{t^{2n-1}\,\tilde\Delta}\Big(dz - \sum_i^n \ell_i\,\tilde\mu_i^2\, d\phi_i\Big)^2\,, \tag{5.4}$$

where

$$\tilde H_i = 1 + \frac{\ell_i^2}{t^2}\,, \qquad \tilde\Delta = \Big(\tilde\mu_0^2 - \sum_{i=1}^n \frac{\tilde\mu_i^2}{\tilde H_i}\Big)\prod_{i=1}^n \tilde H_i\,. \tag{5.5}$$

This solution is invariant under $t \to -t$, $m \to -m$.

In the asymptotic $t \to \pm\infty$ regions, the solution becomes a $D-1$ dimensional universe with hyperbolic spatial slices expanding or contracting at a constant rate, together with a stable $\mathbb{R}$ or S^1 direction. Note that the quantity $t^{2n}\tilde\Delta$ given by

$$t^{2n}\,\tilde\Delta = \Big(1 + \sum_{i=1}^n \frac{\tilde\mu_i^2\,\ell_i^2}{t^2 + \ell_i^2}\Big)\prod_{i=1}^n (t^2 + \ell_i^2)\,, \tag{5.6}$$

is positive definite for non-vanishing ℓ_i. That is, if any of the ℓ_i is zero, then the above quantity will vanish at $t = 0$ and the solution becomes singular. The global structure is largely determined by the function

$$F = -2mt + t^{2n}\prod_{i=1}^n \tilde H_i\,, \tag{5.7}$$

which appears as a factor in the dt^2 term of the metric (5.4). When all the parameters ℓ_i are non-zero, there exists a critical value m_0, for which there is a time t_0 such that $F(t_0) = 0$ and $F'(t_0) = 0$. The values of m_0 and t_0 depend on the ℓ_i in a complicated way. We only present a special case of $\ell_i = \ell$:

$$t_0 = \frac{\ell}{\sqrt{2n-1}}\,, \qquad m_0 = 2^{n-1}\, n^n\, t_0^{2n-1}\,. \tag{5.8}$$

When $|m| < m_0$, the function F is positive definite. It follows that the solution is regular everywhere, bouncing between two Minkowski spacetimes. When $|m| > m_0$, the function F will have two roots t_+ and t_-, between which F becomes negative and the solution is stationary. Although we expect that there is no curvature singularity in this case, the metric develops closed time-like curves. This can be seen, without loss of generality, by examining the $d\phi_1^2$ term of the metric, given by

$$\frac{\tilde\mu_1^2}{t^{2n}\tilde\Delta}\Big[t^{2n+2}\prod_{i=1}^n \tilde H_i + F\,\tilde\mu_1^2\,\ell_i^2 + t^{2n}\Big(\prod_{i=1}^n \tilde H_i\Big)\sum_{i=2}^n \frac{\tilde\mu_i^2\,\ell_i^2}{t^2\,\tilde H_i}\Big]\,. \tag{5.9}$$

Thus, the compact coordinates ϕ_i are always space-like if F is positive but can become time-like if F is negative, since the $\tilde\mu_i$ are not bounded.

Finally, when $|m| = m_0$, the function F has a second-order zero at $t = t_0$ but never becomes negative. Near $t = t_0$, the metric contains an element of two-dimensional de Sitter spacetime. To see this, we can complete the square of $d\phi_i$, and the remaining dz^2 term becomes

$$\frac{\tilde{\Delta}\, F}{\tilde{\Delta}\, F + 2mt \prod_{i=1}^n \tilde{H}_i} \,. \tag{5.10}$$

Thus, the metric components involving the t and z coordinates around $t = t_0$ are given by

$$ds^2 = \tilde{\Delta} \left[\frac{(t-t_0)^2\, F''(t_0)}{4m\, t_0 \prod_{i=1}^n \tilde{H}_i}\, dz^2 - \frac{2t_0^{2n}}{(t-t_0)^2\, F''(t_0)}\, dt^2 \right] , \tag{5.11}$$

where $\tilde{H}_i = 1 + \ell_i^2/t_0^2$. The metric is clearly two-dimensional de Sitter spacetime with $t = t_0$ being the infinite past in the comoving frame. It has a warp factor $\tilde{\Delta}$ that depends on the coordinates of the hyperbolic space. Thus, the D-dimensional metric at $t = t_0$ describes a warp product of dS_2 and a hyperbolic $2n$-space.

To summarize, the S-Kerr solution in even-dimensions is regular, provided that all the ℓ_i parameters are turned on and $|m| \le m_0$. For $|m| < m_0$, the solution describes a smooth bounce between two phases of Minkowski spacetime. For $|m| = m_0$, the solution smoothly runs from a warp product of hyperbolic $2n$-space and two-dimensional de Sitter spacetime in the infinite past to Minkowski spacetime in the infinite future. Lastly, for $|m| > m_0$, there are closed time-like curves.

5.2 Odd dimensions: $D = 2n + 1$

The Kerr-Schild solution in $D = 2n+1$ dimensions, with a sphere of $d = 2n-1$ dimensions, has the metric [24]

$$\begin{aligned} ds_D^2 \;=\; & -dt^2 + \frac{\Delta\, dr^2}{\prod_{i=1}^n H_i - \frac{2m}{r^{2n-2}}} + r^2 \sum_{i=1}^n H_i\, (d\mu_i^2 + \mu_i^2\, d\phi_i^2) + \\ & + \frac{2m}{r^{2n-2}\, \Delta} \Big(dt - \sum_i \ell_i\, \mu_i^2\, d\phi_i \Big)^2 , \end{aligned} \tag{5.12}$$

where

$$\sum_{i=1}^n \mu_i^2 = 1\,, \qquad \Delta = \Big(\sum_{i=1}^n \frac{\mu_i^2}{H_i} \Big) \prod_{i=1}^n H_i\,, \tag{5.13}$$

and $H_i = 1 + \ell_i^2/r^2$. We can now perform the analytical continuation

$$\begin{aligned} & t \to \mathrm{i}\, z\,, \qquad r \to \mathrm{i}\, t\,, \qquad \phi_n \to \mathrm{i}\, \phi_n\,, \qquad \alpha \to \mathrm{i}\, \alpha\,, \\ & m \to -\mathrm{i}\, m\,, \qquad \ell_j \to -\mathrm{i}\, \ell_j\,, \qquad j \;=\; 1, \ldots, n-1\,, \end{aligned}$$

and ℓ_n is unchanged. We define $\mu_n = \tilde{\mu}_n$ and $\mu_j = \mathrm{i}\, \tilde{\mu}_j$, such that $\tilde{\mu}_n^2 - \sum_{j=1}^{n-1} \tilde{\mu}_j^2 = 1$. The Kerr-Schild solution becomes an S-Kerr solution, given by

$$ds_D^2 \;=\; dz^2 - \frac{\tilde{\Delta}\, dt^2}{\prod_{i=1}^n \tilde{H}_i - \frac{2m}{t^{2n-2}}} + t^2 \sum_{i=1}^{n-1} \tilde{H}_i\, (d\tilde{\mu}_i^2 + \tilde{\mu}_i^2\, d\phi_i^2) +$$

$$+\tilde{H}_n\,(-d\tilde{\mu}_n^2+\tilde{\mu}_n^2\,d\phi_n^2)-\frac{2m}{t^{2n-2}\,\tilde{\Delta}}\Big(dz-\sum_i^n \ell_i\,\tilde{\mu}_i^2\,d\phi_i\Big)^2\,, \tag{5.15}$$

where

$$\begin{aligned}\tilde{\Delta}&=\Big(\tilde{\mu}_n^2-\sum_{j=1}^{n-1}\frac{\tilde{\mu}_j^2}{\tilde{H}_j}\Big)\prod_{i=1}^n\tilde{H}_i\,,\\ \tilde{H}_n&=1-\frac{\ell_n^2}{t^2}\,,\qquad \tilde{H}_j=1+\frac{\ell_j^2}{t^2}\,,\qquad j=1,\ldots,n-1\,.\end{aligned} \tag{5.16}$$

This solution is invariant under $t\to -t$. In the asymptotic $t\to\pm\infty$ regions, the metric describes a $D-1$ dimensional universe with hyperbolic spatial slices expanding or contracting at a constant rate, together with a stable $\mathbb{R}$ or S^1 direction. For the metric to be regular, it is necessary that $\ell_n=0$, and all the remaining ℓ_i non-vanishing. This ensures that the term $t^{2n}\tilde{\Delta}$ is positive definite. The global structure is then largely determined by the function

$$F=-2m+t^{2n-2}\prod_{i=1}^{n-1}\tilde{H}_i\,. \tag{5.17}$$

For small m, namely

$$m<m_0=\frac{1}{2}\prod_{i=1}^{n-1}\ell_i^2\,,\qquad \ell_n=0\,, \tag{5.18}$$

the function F is positive definite and the metric is regular everywhere, bouncing between two Minkowski spacetimes. When $m\ge m_0$, the function has two real roots $\pm t_0$, in between which the metric becomes stationary. The metric develops closed time-like curves in this region. This can be seen by looking at the term $d\phi_1^2$, which is given by (5.9) with $\tilde{H}_n=1$ and $\ell_n=0$. Thus, a regular solution emerges only when the condition (5.18) is satisfied with m_0 non-vanishing.

6. Non-singular Sp-branes

Single-charge p-branes are solutions of the lagrangian

$$e^{-1}\,\mathcal{L}_D=R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2\tilde{n}!}e^{a\,\phi}\,(F_{\tilde{n}})^2\,, \tag{6.1}$$

where $F_{(\tilde{n})}=dA_{(\tilde{n}-1)}$ and $a^2=4-2(\tilde{n}-1)(D-\tilde{n}-1)/(D-2)$ [25]. This lagrangian admits an electric $(d-1)$-brane with $d=\tilde{n}-1$ and a magnetic $(d-1)$-brane with $d=D-\tilde{n}-1$. Since the magnetic solution can be considered as an electric solution of the dual $(D-\tilde{n})$-form field strength $F_{(D-\tilde{n})}$, we need only explicitly consider the electric solution. A dual parameter is given by $\tilde{d}=D-d-2$.

The rotating p-brane solutions [21] can be analytically continued to non-singular Sp-branes. Since the calculation is a straightforward extension of the previous cases, here we simply present the resulting Sp-brane solutions. If the foliating hyperbolic space of the transverse space is of even dimension, corresponding to $\tilde{d} = 2n - 1$, then the Sp-brane metric is given by

$$\begin{aligned} ds_D^2 = {} & \tilde{H}^{-\frac{\tilde{d}}{D-2}} \left[\left(1 - \frac{2m}{t^{\tilde{d}}\, \tilde{\Delta}}\right) dz^2 + dx_1^2 + \cdots + dx_p^2 \right] + \\ & + \tilde{H}^{\frac{d}{D-2}} \Bigg[- \frac{\tilde{\Delta}\, dt^2}{\prod_{i=1}^n \tilde{H}_i - \frac{2m}{t^{\tilde{d}}}} + t^2 \left(- d\tilde{\mu}_0^2 + \sum_{i=1}^n (d\tilde{\mu}_i^2 + \tilde{\mu}_i^2\, d\phi_i^2) \right) + \\ & \qquad + \frac{4m \cos\alpha}{t^{\tilde{d}}\, \tilde{H}\, \tilde{\Delta}}\, dz \left(\sum_{i=1}^n \ell_i\, \tilde{\mu}_i^2\, d\phi_i \right) - \frac{2m}{t^{\tilde{d}}\, \tilde{H}\, \tilde{\Delta}} \left(\sum_{i=1}^n \ell_i\, \tilde{\mu}_i^2\, d\phi_i \right)^2 \Bigg], \end{aligned} \tag{6.2}$$

and the dilaton ϕ and gauge potential $A_{(n-1)}$ are given by

$$e^{2\phi/a} = \tilde{H}\,, \qquad A_{(n-1)} = \frac{1 - \tilde{H}^{-1}}{\sin\alpha} \left(\cos\alpha\, dz + \sum_{i=1}^n \ell_i\, \tilde{\mu}_i^2\, d\phi_i \right) \wedge d^{n-2}x\,, \tag{6.3}$$

where

$$\tilde{\Delta} = \prod_{i=1}^n \tilde{H}_i \left(\tilde{\mu}_0^2 - \sum_{i=1}^n \frac{\tilde{\mu}_i^2}{\tilde{H}_i} \right), \qquad \tilde{H} = 1 - \frac{2m \sin^2\alpha}{t^{\tilde{d}}\, \tilde{\Delta}}\,, \qquad \tilde{H}_i = 1 + \frac{\ell_i^2}{t^2}\,. \tag{6.4}$$

The $\tilde{\mu}_i$ satisfy $\tilde{\mu}_0^2 - \sum_{i=1}^n \tilde{\mu}_i^2$. This solution is invariant under $t \to -t$, $m \to (-1)^{\tilde{d}}\, m$.

Since the analysis of the regularity and geometry are similar to that of the previous solutions, we will not repeat it here. The main point is that there is a finite m_0 such that, for $m < m_0$ (more precisely, $|m| < m_0$ in the case of odd $\tilde{d}$), the geometry is regular and exhibits a smooth bounce between two phases of Minkowski spacetime. For $m = m_0$, the geometry runs from a warped product of two-dimensional de Sitter spacetime, p-dimensional euclidean space and a hyperbolic $2n$-space in the infinite past to Minkowski spacetime in the infinite future. The warp factor depends on the hyperbolic coordinates. Finally, for $m > m_0$, there are closed time-like curves and, for $m \geq m_0/\sin^2\alpha$, a curvature singularity.

If the foliating hyperbolic space of the transverse space is of odd dimension, then the solution is given by (6.2) with $\tilde{d} = 2n$ and $\tilde{\mu}_0 = 0$. The singularity structure of the geometry is similar to above except that, instead of having a de Sitter component at $m = m_0$, there are closed time-like curves.

7. Conclusions

We have found twisted Sp-brane and S-Kerr solutions by analytically continuing rotating p-branes and higher-dimensional Kerr black holes, respectively. There is always a region in parameter space for which these time-dependent solutions are completely regular, even though the corresponding static solutions are marred by a naked singularity. This is among a growing number of examples for which singularities of static black hole or p-brane solutions can be analytically continued to extend onto a smooth manifold.

The precise structure of these time-dependent solutions depends on whether the foliating hyperbolic space of the transverse space is odd or even-dimensional. In the former case, if a single ℓ_i vanishes, then for the range $m < m_0$ there is a smooth bounce between two phases of Minkowski spacetime. This case includes twisted SM2 and SD3-branes. On the other hand, for an even-dimensional foliating hyperbolic space, a smooth bounce between two phases of Minkowski spacetime is exhibited within the range $|m| < m_0$, provided that none of the ℓ_i vanish. Also, for $|m| = m_0$, the solution smoothly runs from a warped product containing two-dimensional de Sitter spacetime, for which the warp factor depends on hyperbolic coordinates, to Minkowski spacetime. This latter case includes the twisted SM5-brane. Regardless of the dimensionality of the foliating hyperbolic space, for $m \geq m_0$ there are closed time-like curves. In the presence of a non-zero brane charge parameter α, there is a curvature singularity for $m \geq m_0/\sin^2\alpha$.

Although some of these solutions exhibit a two-dimensional de Sitter phase, it would naturally be quite nice to find solutions for which the spacetime element which undergoes exponential expansion has four dimensions. Therefore, it is of substantial interest to uncover a greater array of regular time-dependent solutions, and taking the analytical continuation of static solutions provides a guiding light in this endeavor.

The time evolution of our solutions may be used to investigate particle production in the early universe, the geometrical backreaction of tachyon condensation, and may provide regular supergravity backgrounds on which to study the large-N open-closed string duality in a time-dependent context.

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